Menkar
Towards a Multimode Presheaf Proof Assistant

Andreas Nuyts[1], Dominique Devriese[2]

[1]KU Leuven, Belgium
[2]Vrije Universiteit Brussel, Belgium

TYPES '19
Oslo, Norway
June 13, 2019

Modal type theory:
Examples

Irrelevance

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\, X$,
- $cons_X : (\mathbf{irr} \mid m\, n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\, X \to \mathbf{List}_n\, X$

$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N}$ OK: $(\mathbf{rel} \geq \mathbf{rel})$

$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N}$ OK: $(\mathbf{rel} \geq \mathbf{rel})$

$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n$ OK: $(\mathbf{rel} \geq \mathbf{rel})$

$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X$ OK: $(\mathbf{rel} \geq \mathbf{rel})$

$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\, X$ BAD: $(\mathbf{rel} \not\geq \mathbf{irr})$

$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n\, X$

$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel}$ $\mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$

$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr}$ $\mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\, X,$
- $cons_X : (\mathbf{irr} \mid m\, n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\, X \to \mathbf{List}_n\, X$

$$\frac{\begin{array}{ll}(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\, X & \textbf{BAD: } (\mathbf{rel} \not\geq \mathbf{irr})\end{array}}{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n\, X}$$

$$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel} \qquad \mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$$
$$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr} \qquad \mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$$

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\,x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\,X,$
- $cons_X : (\mathbf{irr} \mid m\,n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\,X \to \mathbf{List}_n\,X$

$$\frac{\begin{array}{ll} (\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} & \text{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} & \text{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n & \text{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X & \text{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\ (\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\,X & \text{BAD: } (\mathbf{rel} \not\geq \mathbf{irr}) \end{array}}{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n\,X}$$

$$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel} \qquad \mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$$
$$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr} \qquad \mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$$

Irrelevance

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\textbf{irr} \mid n : \mathbb{N}) \to (\textbf{irr} \mid 0 < n) \to \textbf{List}_n\, X,$
- $cons_X : (\textbf{irr} \mid m\, n : \mathbb{N}) \to (\textbf{irr} \mid m < n) \to X \to \textbf{List}_m\, X \to \textbf{List}_n\, X$

$$\frac{\begin{array}{ll} (\textbf{rel} \mid m)(\textbf{rel} \mid n)(\textbf{rel} \mid p)(\textbf{rel} \mid x)(\textbf{rel} \mid xs) \vdash m : \mathbb{N} & \textbf{OK: } (\textbf{rel} \geq \textbf{rel}) \\ (\textbf{rel} \mid m)(\textbf{rel} \mid n)(\textbf{rel} \mid p)(\textbf{rel} \mid x)(\textbf{rel} \mid xs) \vdash n : \mathbb{N} & \textbf{OK: } (\textbf{rel} \geq \textbf{rel}) \\ (\textbf{rel} \mid m)(\textbf{rel} \mid n)(\textbf{rel} \mid p)(\textbf{rel} \mid x)(\textbf{rel} \mid xs) \vdash p : m < n & \textbf{OK: } (\textbf{rel} \geq \textbf{rel}) \\ (\textbf{rel} \mid m)(\textbf{irr} \mid n)(\textbf{irr} \mid p)(\textbf{rel} \mid x)(\textbf{irr} \mid xs) \vdash x : X & \textbf{OK: } (\textbf{rel} \geq \textbf{rel}) \\ (\textbf{rel} \mid m)(\textbf{irr} \mid n)(\textbf{irr} \mid p)(\textbf{rel} \mid x)(\textbf{irr} \mid xs) \vdash xs : \textbf{List}_m\, X & \textbf{BAD: } (\textbf{rel} \not\geq \textbf{irr}) \end{array}}{(\textbf{rel} \mid m)(\textbf{irr} \mid n)(\textbf{irr} \mid p)(\textbf{rel} \mid x)(\textbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \textbf{List}_n\, X}$$

$$\begin{array}{ll} \textbf{rel} \setminus \textbf{rel} = \textbf{rel} & \textbf{irr} \setminus \textbf{rel} = \textbf{rel} \\ \textbf{rel} \setminus \textbf{irr} = \textbf{irr} & \textbf{irr} \setminus \textbf{irr} = \textbf{rel} \end{array}$$

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n X,$
- $cons_X : (\mathbf{irr} \mid m\, n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m X \to \mathbf{List}_n X$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m X \qquad \textbf{BAD: } (\mathbf{rel} \not\geq \mathbf{irr})$$

$$\overline{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n X}$$

$$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel} \qquad \mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$$

$$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr} \qquad \mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$$

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\,x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\,X$,
- $cons_X : (\mathbf{irr} \mid m\,n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\,X \to \mathbf{List}_n\,X$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X \qquad \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel})$$

$$\frac{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\,X \qquad \textbf{BAD: } (\mathbf{rel} \not\geq \mathbf{irr})}{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\; m\; n\; p\; x\; xs : \mathbf{List}_n\,X}$$

$$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel} \qquad \mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$$

$$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr} \qquad \mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$$

Irrelevance

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\, X,$
- $cons_X : (\mathbf{irr} \mid m\, n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\, X \to \mathbf{List}_n\, X$

$$\frac{\begin{array}{ll}
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\, X & \textbf{BAD: } (\mathbf{rel} \not\geq \mathbf{irr})
\end{array}}{(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n\, X}$$

$\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel}$ $\quad$ $\mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel}$

$\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr}$ $\quad$ $\mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}$

Irrelevance: dependent definitional constancy

Agda notation: $.(x : A) \to B\ x$

Sized lists:

- $nil_X : (\mathbf{irr} \mid n : \mathbb{N}) \to (\mathbf{irr} \mid 0 < n) \to \mathbf{List}_n\, X,$
- $cons_X : (\mathbf{irr} \mid m\, n : \mathbb{N}) \to (\mathbf{irr} \mid m < n) \to X \to \mathbf{List}_m\, X \to \mathbf{List}_n\, X$

$$
\frac{
\begin{array}{ll}
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash m : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash n : \mathbb{N} & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{rel} \mid n)(\mathbf{rel} \mid p)(\mathbf{rel} \mid x)(\mathbf{rel} \mid xs) \vdash p : m < n & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash x : X & \textbf{OK: } (\mathbf{rel} \geq \mathbf{rel}) \\
(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash xs : \mathbf{List}_m\, X & \textbf{BAD: } (\mathbf{rel} \not\geq \mathbf{irr})
\end{array}
}{
(\mathbf{rel} \mid m)(\mathbf{irr} \mid n)(\mathbf{irr} \mid p)(\mathbf{rel} \mid x)(\mathbf{irr} \mid xs) \vdash cons_X\ m\ n\ p\ x\ xs : \mathbf{List}_n\, X
}
$$

$$
\begin{array}{ll}
\mathbf{rel} \setminus \mathbf{rel} = \mathbf{rel} & \mathbf{irr} \setminus \mathbf{rel} = \mathbf{rel} \\
\mathbf{rel} \setminus \mathbf{irr} = \mathbf{irr} & \mathbf{irr} \setminus \mathbf{irr} = \mathbf{rel}
\end{array}
$$

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\textbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\textbf{cri} \mid \mathcal{U}) \to \mathcal{U}$
  Licata, Orton, Pitts, Spitters (2018)

$$\dfrac{\dfrac{\begin{array}{ll}(\textbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} & (\textbf{con} \geq \textbf{cri}) \\ (\textbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U} & \text{BAD: unavailable}\end{array}}{(\textbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\textbf{cri} \mid A : \mathcal{U})(\textbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$$\textbf{cri} \setminus \textbf{cri} = \textbf{cri} \qquad \textbf{con} \setminus \textbf{cri} = \textbf{cri}$$
$$\textbf{cri} \setminus \textbf{con} = \varnothing \qquad \textbf{con} \setminus \textbf{con} = \textbf{con}$$

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\mathbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\mathbf{cri} \mid \mathcal{U}) \to \mathcal{U}$
  Licata, Orton, Pitts, Spitters (2018)

$$\dfrac{\dfrac{\begin{array}{ll} \vdash A : \mathcal{U} & (\mathbf{con} \geq \mathbf{cri}) \\ \vdash B : \mathcal{U} & \text{BAD: unavailable} \end{array}}{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\mathbf{cri} \mid A : \mathcal{U})(\mathbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$$\mathbf{cri} \setminus \mathbf{cri} = \mathbf{cri} \qquad \mathbf{con} \setminus \mathbf{cri} = \mathbf{cri}$$

$$\mathbf{cri} \setminus \mathbf{con} = \emptyset \qquad \mathbf{con} \setminus \mathbf{con} = \mathbf{con}$$

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\mathbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\mathbf{cri} \mid \mathcal{U}) \to \mathcal{U}$

  Licata, Orton, Pitts, Spitters (2018)

$(\mathbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U}$ OK: $(\mathbf{con} \geq \mathbf{cri})$

$(\mathbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U}$ BAD: unavailable

$$\dfrac{\dfrac{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} \qquad (\mathbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U}}{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\mathbf{cri} \mid A : \mathcal{U})(\mathbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$\mathbf{cri} \setminus \mathbf{cri} = \mathbf{cri}$ $\qquad$ $\mathbf{con} \setminus \mathbf{cri} = \mathbf{cri}$

$\mathbf{cri} \setminus \mathbf{con} = \varnothing$ $\qquad$ $\mathbf{con} \setminus \mathbf{con} = \mathbf{con}$

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\textbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\textbf{cri} \mid \mathcal{U}) \to \mathcal{U}$
  Licata, Orton, Pitts, Spitters (2018)

$(\textbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U}$ **OK:** $(\textbf{con} \geq \textbf{cri})$

$(\textbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U}$ **BAD:** unavailable

$$\frac{\dfrac{(\textbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} \qquad (\textbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U}}{(\textbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\textbf{cri} \mid A : \mathcal{U})(\textbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$\textbf{cri} \setminus \textbf{cri} = \textbf{cri}$ $\qquad$ $\textbf{con} \setminus \textbf{cri} = \textbf{cri}$

$\textbf{cri} \setminus \textbf{con} = \oslash$ $\qquad$ $\textbf{con} \setminus \textbf{con} = \textbf{con}$

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\mathbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\mathbf{cri} \mid \mathcal{U}) \to \mathcal{U}$

  Licata, Orton, Pitts, Spitters (2018)

$$\dfrac{\dfrac{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} \quad \text{OK: } (\mathbf{con} \geq \mathbf{cri}) \qquad (\mathbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U} \quad \text{BAD: unavailable}}{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\mathbf{cri} \mid A : \mathcal{U})(\mathbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$$\mathbf{cri} \setminus \mathbf{cri} = \mathbf{cri} \qquad \mathbf{con} \setminus \mathbf{cri} = \mathbf{cri}$$

$$\mathbf{cri} \setminus \mathbf{con} = \emptyset \qquad \mathbf{con} \setminus \mathbf{con} = \mathbf{con}$$

Crispness (HoTT)

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\textbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\textbf{cri} \mid \mathcal{U}) \to \mathcal{U}$

  Licata, Orton, Pitts, Spitters (2018)

$$\dfrac{\dfrac{\begin{array}{ll}(\textbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} & \textbf{OK: } (\textbf{con} \geq \textbf{cri}) \\ (\textbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U} & \textbf{BAD: } \text{unavailable}\end{array}}{(\textbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\textbf{cri} \mid A : \mathcal{U})(\textbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$\textbf{cri} \setminus \textbf{cri} = \textbf{cri}$ $\qquad$ $\textbf{con} \setminus \textbf{cri} = \textbf{cri}$

$\textbf{cri} \setminus \textbf{con} = \emptyset$ $\qquad$ $\textbf{con} \setminus \textbf{con} = \textbf{con}$

Crispness (HoTT)

Crisp functions do not preserve paths.

Agda-flat notation: $(x :\{\flat\}\ A) \to B\ x$

- Strict equality $\_ \doteq \_ : (\mathbf{cri} \mid x\ y : A) \to \mathcal{U}$
- "Amazing right adjoint" $\surd : (\mathbf{cri} \mid \mathcal{U}) \to \mathcal{U}$

  Licata, Orton, Pitts, Spitters (2018)

$$\dfrac{\dfrac{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A : \mathcal{U} \quad \textbf{OK:}\ (\mathbf{con} \geq \mathbf{cri}) \qquad (\mathbf{cri} \mid A : \mathcal{U}) \vdash B : \mathcal{U} \quad \textbf{BAD:}\ \text{unavailable}}{(\mathbf{cri} \mid A : \mathcal{U}) \vdash A \times B : \mathcal{U}}}{(\mathbf{cri} \mid A : \mathcal{U})(\mathbf{con} \mid B : \mathcal{U}) \vdash \surd(A \times B) : \mathcal{U}}$$

$$\mathbf{cri} \setminus \mathbf{cri} = \mathbf{cri} \qquad \mathbf{con} \setminus \mathbf{cri} = \mathbf{cri}$$

$$\mathbf{cri} \setminus \mathbf{con} = \oslash \qquad \mathbf{con} \setminus \mathbf{con} = \mathbf{con}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

  $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\, x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\, t : B\, t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}^p_\ell$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

  $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\, x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\, t : B\, t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\,x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\,t : B\,t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}^p_\ell$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\,x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\,t : B\,t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$
    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$
  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$
    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \backslash - \dashv \mu \circ -$

  $\mu \backslash \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\,x \qquad \mu \backslash \Gamma \vdash t : A}{\Gamma \vdash f\,t : B\,t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}^p_\ell$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \backslash - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\,x \qquad \mu \backslash \Gamma \vdash_p t : A}{\Gamma \vdash_q f\,t : B\,t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\,x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\,t : B\,t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\,x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\,t : B\,t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$
    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$
  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$
    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \dots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \dots$
- Types $A : \mathcal{U}^p_\ell$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\, x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\, t : B\, t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}_\ell^p$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\, x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\, t : B\, t}$$

The general principle

Modal type theory

- Poset of modalities $\mu, \nu, \rho, \ldots$
- Functions have a modality
  - Modality ε of $\lambda x.x$
  - Modality $\nu \circ \mu$ of $g \circ f$

    $\Rightarrow$ ordered monoid
- Left division $\mu \setminus - \dashv \mu \circ -$

  $\mu \setminus \rho \leq \nu \Leftrightarrow \rho \leq \mu \circ \nu$

Pioneered by:

Pfenning (2001), Abel (2006, 2008)

$$\frac{\Gamma \vdash f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash t : A}{\Gamma \vdash f\ t : B\ t}$$

Multimode type theory

- Set of modes $p, q, r, \ldots$
- Types $A : \mathcal{U}^p_\ell$ have a mode
- Posets of mod'ties $\mu : p \to q$
- Functions have a modality
  - $\mathrm{id}_p : p \to p$
  - $- \circ -$

    $\Rightarrow$ poset-enriched category
- Left division $\mu \setminus - \dashv \mu \circ -$

Licata, Shulman (2016)

Licata, Shulman, Riley (2017)

$$\frac{\mu : p \to q \qquad \Gamma \vdash_q f : (\mu \mid x : A) \to B\ x \qquad \mu \setminus \Gamma \vdash_p t : A}{\Gamma \vdash_q f\ t : B\ t}$$

System Fω

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\mathbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \mathbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \oslash$$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathbf{par} = \mathbf{par} \qquad \mathbf{par} \setminus \mathbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\dfrac{\begin{array}{c}(\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} b : \mathbf{Bool} \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} y : X\end{array}}{(\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\textbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \textbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \oslash$$

$$\mathrm{id}_{\mathtt{data}} \setminus \textbf{par} = \textbf{par} \qquad \textbf{par} \setminus \textbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\frac{\begin{array}{l} (\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\textbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \textbf{Bool}) \vdash_{\mathtt{data}} b : \textbf{Bool} \\ (\textbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \textbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\textbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \textbf{Bool}) \vdash_{\mathtt{data}} y : X \end{array}}{(\textbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \textbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

System Fω

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\mathbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \mathbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \oslash$$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathbf{par} = \mathbf{par} \qquad \mathbf{par} \setminus \mathbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\frac{\begin{array}{c}(\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} b : \mathbf{Bool} \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} y : X\end{array}}{(\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\mathbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \mathbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \emptyset$$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathbf{par} = \mathbf{par} \qquad \mathbf{par} \setminus \mathbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\frac{\begin{array}{c}(\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} b : \mathbf{Bool} \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} y : X\end{array}}{(\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\mathbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \mathbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \oslash$$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathbf{par} = \mathbf{par} \qquad \mathbf{par} \setminus \mathbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\frac{\begin{array}{c}(\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} b : \mathbf{Bool} \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} y : X\end{array}}{(\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

- Modes: $\mathtt{data}, \mathtt{type}$
- Modalities:
  - $\mathrm{id}_{\mathtt{data}} : \mathtt{data} \to \mathtt{data}$
  - $\mathbf{par} : \mathtt{type} \to \mathtt{data}$
  - $\mathrm{id}_{\mathtt{type}} : \mathtt{type} \to \mathtt{type}$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathrm{id}_{\mathtt{data}} = \mathrm{id}_{\mathtt{data}} \qquad \mathbf{par} \setminus \mathrm{id}_{\mathtt{data}} = \oslash$$

$$\mathrm{id}_{\mathtt{data}} \setminus \mathbf{par} = \mathbf{par} \qquad \mathbf{par} \setminus \mathbf{par} = \mathrm{id}_{\mathtt{type}}$$

$$\mathrm{id}_{\mathtt{type}} \setminus \mathrm{id}_{\mathtt{type}} = \mathrm{id}_{\mathtt{type}}$$

$$\dfrac{\begin{array}{l} (\mathrm{id}_{\mathtt{type}} \mid X : *) \vdash_{\mathtt{type}} X : * \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} b : \mathbf{Bool} \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} x : X \\ (\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} y : X \end{array}}{(\mathbf{par} \mid X : *)(\mathrm{id}_{\mathtt{data}} \mid x, y : X)(\mathrm{id}_{\mathtt{data}} \mid b : \mathbf{Bool}) \vdash_{\mathtt{data}} \mathit{if}\ X\ b\ x\ y : X}$$

Degrees of Relatedness

Nuyts, Devriese (2018)

- ∞ **modes:** `proof`, `data`, `type`, `kind`, `sort`, ...
- ∞ **modalities** incl. ad hoc polymorphism, continuity, parametricity, structurality, irrelevance, shape-irrelevance **at every mode**.

⇒ Superhuman bookkeeping task: Need type-checker.

⇒ Code duplication for every modality is **impossible**:
Need mode & modality polymorphism.

Degrees of Relatedness

Nuyts, Devriese (2018)

- ∞ **modes:** `proof`, `data`, `type`, `kind`, `sort`, ...
- ∞ **modalities** incl. ad hoc polymorphism, continuity, parametricity, structurality, irrelevance, shape-irrelevance **at every mode**.

⇒ Superhuman bookkeeping task: Need type-checker.

⇒ Code duplication for every modality is **impossible**:
Need mode & modality polymorphism.

Degrees of Relatedness

Nuyts, Devriese (2018)

- ∞ **modes:** `proof`, `data`, `type`, `kind`, `sort`, ...
- ∞ **modalities** incl. ad hoc polymorphism, continuity, parametricity, structurality, irrelevance, shape-irrelevance **at every mode**.

$\Rightarrow$ Superhuman bookkeeping task: Need type-checker.

$\Rightarrow$ Code duplication for every modality is **impossible**:
Need mode & modality polymorphism.

Degrees of Relatedness

Nuyts, Devriese (2018)

- ∞ **modes:** `proof`, `data`, `type`, `kind`, `sort`, ...
- ∞ **modalities** incl. ad hoc polymorphism, continuity, parametricity, structurality, irrelevance, shape-irrelevance **at every mode**.

$\Rightarrow$ Superhuman bookkeeping task: Need type-checker.

$\Rightarrow$ Code duplication for every modality is **impossible**:
Need mode & modality polymorphism.

Degrees of Relatedness

Nuyts, Devriese (2018)

- ∞ **modes:** `proof`, `data`, `type`, `kind`, `sort`, ...
- ∞ **modalities** incl. ad hoc polymorphism, continuity, parametricity, structurality, irrelevance, shape-irrelevance **at every mode**.

$\Rightarrow$ Superhuman bookkeeping task: Need type-checker.

$\Rightarrow$ Code duplication for every modality is **impossible**: Need mode & modality polymorphism.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^{μ}, Σ^{μ}, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi\ ?\ a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^μ, Σ^μ, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi \mathbin{?} a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^μ, Σ^μ, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi \mathbin{?} a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathsf{HS}}$ (object classifier), Π^μ, Σ^μ, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi ? a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^{μ}, Σ^{μ}, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi \mathrel{?} a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^{μ}, Σ^{μ}, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi\ ?\ a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^{μ}, Σ^{μ}, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi ? a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Goals of Menkar

A **multimode** proof assistant:

- General rules of multimode* TT hardcoded
- Specific multimode systems to be written in **pluggable modules**
- Support for internal mode/modality polymorphism*

A **presheaf** proof assistant:

- Primitives for:
  - $\mathcal{U}^{\mathrm{HS}}$ (object classifier), Π^{μ}, Σ^{μ}, strict equality, ...
  - **Prop** (subobject classifier), $\vee$, $\wedge$, $\top$, $\bot$, ...
  - Extension types $A[\varphi\ ?\ a]$
  - Orton & Pitts's **strictness axiom**
  - **Dependable atomicity*** (cf. my talk tomorrow)
- System-specific: $\mathbb{I}$, $(\_ = 0)$, $(\_ = 1)$, ...
- Should be definable:
  Glue, Weld, Ψ ("relativity") (cf. G. Moulin's PhD, 2016)

* Infeasible to extend existing proof assistant.

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors,
  Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

Intended use

Pen & paper / LaTeX

- Define cat. of modes $\mathcal{P}$,
- Pick (presheaf) models: $\mathcal{F} : \mathcal{P} \to \mathrm{CwF}$,
- Check requirements of presheaf primitives,
- Model custom primitives.

In Haskell

- Add syntax, reduction and typing rules
  - for modalities,
  - for custom primitives,
- Implement id and $- \circ -$.

In Menkar

- Define fibrancy,
- Construct a fibrant universe,
- Prove fibrancy of type constructors, Planned: Instance arguments
- Have modal fun!

State of the project:

- Type-checks DTT with 1 mode and 1 modality
- Degrees of Relatedness in progress…

Possible applications:

- Cubes for parametricity
- Cubes for HoTT
- Guarded type theory with clock irrelevance and time warps?
  Time warps: Adrien Guatto (2018)
- Directed degrees of relatedness
- …

State of the project:

- Type-checks DTT with 1 mode and 1 modality
- Degrees of Relatedness in progress...

Possible applications:

- Cubes for parametricity
- Cubes for HoTT
- Guarded type theory with clock irrelevance and time warps?
  Time warps: Adrien Guatto (2018)
- Directed degrees of relatedness
- ...

State of the project:

- Type-checks DTT with 1 mode and 1 modality
- Degrees of Relatedness in progress...

Possible applications:

- Cubes for parametricity
- Cubes for HoTT
- Guarded type theory with clock irrelevance and time warps?
  Time warps: Adrien Guatto (2018)
- Directed degrees of relatedness
- ...

State of the project:

- Type-checks DTT with 1 mode and 1 modality
- Degrees of Relatedness in progress...

Possible applications:

- Cubes for parametricity
- Cubes for HoTT
- Guarded type theory with clock irrelevance and time warps?
  Time warps: Adrien Guatto (2018)
- Directed degrees of relatedness
- ...

State of the project:

- Type-checks DTT with 1 mode and 1 modality
- Degrees of Relatedness in progress...

Possible applications:

- Cubes for parametricity
- Cubes for HoTT
- Guarded type theory with clock irrelevance and time warps?
  Time warps: Adrien Guatto (2018)
- Directed degrees of relatedness
- ...

Take home message

If you need a **multimode/presheaf proof assistant**,
don't hesitate to get in touch :-)

`https://github.com/anuyts/menkar`

Thanks!

Questions?